FINAL REPORT FOR THE UNMANNED, SPACE-BASED, REUSABLE ORBITAL TRANSFER VEHICLE "DARVES"
Volume II: Data and Calculations

A design project by students in the Department of Aerospace Engineering at Auburn University, Auburn, Alabama, under the sponsorship of NASA/USRA Advanced Design Program.

Auburn University
Auburn, Alabama
June, 1988

CONTRIBUTORS

Scott H. Kester ------------------- Structures

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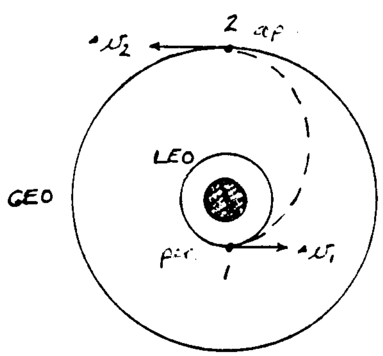
Susan J. Spry --------------------- Trajectories, Aerobrake

Tracey L. Touchton ---------------- Propulsion

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Sample Trajectory Analysis
LEO → GEO



R_e = 6378.145 km
= 3963.195563 mi
= 2.092567257×10^7 ft
(mean equatorial radius)

M_e = 5.976×10^{24} kg
= 4.095×10^{23} $lbf \cdot s^2/ft$
(mass of earth)

G = 6.673×10^{-11} $m^3/kg \cdot s^2$
= 3.439×10^{-8} $ft^4/lbf \cdot s^4$
(universal grav. const.)

μ_e = 3.986012×10^5 km^3/s^2
= $1.407646882 \times 10^{16}$ ft^3/s^2
(grav. parameter)

LEO
h_1 = 220-270 naut mi = 311 mi from surface of earth at 28.7° inclin.

GEO
h_2 = 35786 km = 22236 mi from surface of earth

no plan chg

from LEO → GEO : (reverse direction for return)

$$v_{1\,circ} = \sqrt{\frac{\mu}{r_1 \text{ from center of earth}}} \doteq \sqrt{\frac{\mu}{(R_e + h_1)}} = \sqrt{\frac{(1.407646882 \times 10^{16} \frac{ft^3}{s^2})(\frac{1\,mi}{5280\,ft})}{(3963.195563 + 311)\,mi}}$$

$$= \left(24{,}974.84\ \frac{ft}{s}\right)\left(\frac{1\ mi}{5280\ ft}\right)\left(\frac{3600\ s}{1\ hr}\right) = 17{,}028.30\ mph$$

$$v_{1\,per} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} \quad \text{where } a_T = \tfrac{1}{2}(r_1 + r_2)$$

$$a_T = \tfrac{1}{2}(R_e + h_1 + R_e + h_2) = \tfrac{1}{2}(2R_e + h_1 + h_2)$$

$$= \tfrac{1}{2}\left(2(3963.195563\ mi) + 311\ mi + 22236\ mi\right) = 15236.70\ mi$$

$$\doteq \sqrt{\mu\left(\frac{2}{R_e + h_1} - \frac{1}{a_T}\right)}$$

$$= \sqrt{\left(1.407646882 \times 10^{16}\ \frac{ft^3}{s^2}\right)\left(\frac{1\ mi}{5280\ ft}\right)\left[\frac{2}{(3963.195563 + 311)\,mi} - \frac{1}{15236.70\ mi}\right]}$$

$$= 32{,}749.23\ \frac{ft}{s} = 22{,}329.02\ mph$$

$$\Delta v_1 = v_{1\,per} - v_{1\,circ} = (22329.02 - 17028.30)\,mph = 5{,}300.72\ mph$$

cont.

$$v_{2ap} = \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a_T}\right)} = \sqrt{\mu\left(\frac{2}{R_e+h_2} - \frac{1}{a_T}\right)}$$

$$= \sqrt{\left(1.407646882\times10^{16}\,\frac{ft^3}{s^2}\right)\left(\frac{1\,mi}{5280\,ft}\right)\left[\frac{2}{(3963.20+22236)\,mi} - \frac{1}{15236.70\,mi}\right]}$$

$$= 5{,}342.79\ \frac{ft}{s} = 3{,}642.81\ mph$$

$$v_{2arc} = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{\mu}{R_e+h_2}} = \sqrt{\frac{1.407646882\times10^{16}\,\frac{ft^3}{s^2}}{(3963.20+22236)\,mi\left(\frac{5280\,ft}{1\,mi}\right)}}$$

$$= 10{,}087.56\ \frac{ft}{s} = 6{,}877.88\ mph$$

$$\Delta v_2 = v_{2ap} - v_{2arc} = (3642.81 - 6877.88)\,mph = -3{,}235.07\ mph$$

Fuel analysis

$$\Delta v = I_{sp}\, g_c \ln\left(\frac{m_i}{m_f}\right)$$ → solve for m_i

$$m_i = m_f\, e^{\frac{\Delta v}{I_{sp}\, g_c}}$$ → g_0 ?

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analyze mission "backwards" w/ the first value for m_f being the dry weight of the OTV.

I_{sp} = 481 s for the RS-44
= 460 s for the RL-10 2B

m_{dry} (OTV w/out payload) = 6000 lb.

$$g_0 = \left(32.2\,\frac{ft}{s^2}\right)\left(\frac{1\,mi}{5280\,ft}\right)\left(\frac{3600\,s}{1\,hr}\right) = 21.9545\ \frac{mi}{hr\cdot s}$$

payload scenarios: 4000; 5000; 10,000 lb.

Case:

1. RS-44 ; 10,000 lb payload

dry OTV in LEO → OTV in elliptical transfer orbit at perigee (for an all-propulsive mission)

$$m_i = m_f\, e^{\frac{\Delta v_1}{I_{sp}\, g_0}}$$

$$= (6000\,lb)\, e^{\left(\frac{5300.72\,\frac{mi}{hr}}{(481\,s)\left(21.9545\,\frac{mi}{hr\cdot s}\right)}\right)} = 9911.6924\ lb$$

$m_f' = 9911.6924$ lb

cont.

OTV in elliptical transfer orbit at apogee → OTV in GEO w/out payload

$$m_i = m_f' e^{\frac{\Delta V_2}{I_{sp} g_o}}$$

$$= (9911.6924 \text{ lb}) e^{\left(\frac{3235.07}{(481)(21.9545)}\right)} = 13{,}464.5793 \text{ lb}$$

$$m_f'' = 13{,}464.5793 \text{ lb}$$

OTV in GEO w/out payload → OTV in GEO w/ payload

$$m_f''' = m_f'' + \text{payload}$$

$$= (13{,}464.5793 + 10{,}000) \text{ lb} = 23{,}464.5793 \text{ lb}$$

OTV in GEO w/ payload → OTV in elliptical transfer orbit at apogee

$$m_i = m_f''' e^{\frac{\Delta V_2}{I_{sp} g_o}} =$$

$$= (23{,}464.5793 \text{ lb}) e^{\left(\frac{3235.07}{(481)(21.9545)}\right)} = 31{,}875.5541 \text{ lb.}$$

$$m_f^{IV} = 31{,}875.5541 \text{ lb}$$

OTV in elliptical transfer orbit at perigee → LEO w/ payload

$$m_i = m_f^{IV} e^{\frac{\Delta V_1}{I_{sp} g_o}}$$

$$= (31{,}875.5541 \text{ lb}) e^{\left(\frac{5500.72}{(481)(21.9545)}\right)} = 52{,}656.78 \text{ lb}$$

∴ initial mass of OTV in LEO w/ payload & full tank

$$= 52{,}656.78 \text{ lb}$$

∴ mass of fuel for 10,000 lb. payload & RS-44 engine

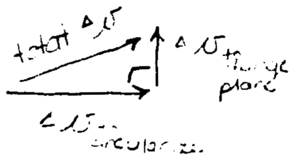
$$m_{fuel} = (52{,}656.78 - 6000 - 10000)$$

$$= \boxed{36{,}656.78 \text{ lb}} \quad \text{all-propulsive}$$

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Simple Plane Change Calculation

Need to incorporate 28.5° plane change Δv at apogee of elliptical transfer orbit (at instant after perform Δv to circularize orbit into GEO) to get final orbit in 0° inclined GEO.



$v_{apo} = 3642.81$ mph

$\Delta v_{\text{to circularize from transfer orbit to GEO}} = 3235.07$ mph

$v_{GEO} = 6877.88$ mph

$\theta = 28.5°$

$\Delta v_{\text{to change plane}}$ does not change final velocity, $v_i = v_f$; only changes plane

$$\Delta v = 2 v \sin\frac{\theta}{2} \quad \text{for circular orbits}$$

must perform plane change at instant after perform Δv to circularize orbit since equation only good for circular orbits.

$$\therefore \Delta v = 2(6877.88 \text{ mph}) \sin\frac{28.5°}{2} = 3386.03 \text{ mph}$$

so that total Δv at apogee of elliptical transfer orbit is found from Pythagoras's theorem since the Δv to change planes is ⊥ to Δv to circularize into GEO.

$$\Delta v_2 = \sqrt{(\Delta v_{\text{to change plane}})^2 + (\Delta v_{\text{to circularize into GEO}})^2}$$

$$= \sqrt{(3386.03)^2 + (3235.07)^2} = 4683.07 \text{ mph}$$

```
rogram FuelMassCalculation;

onst
  At = 80449776.0;          {ft}
  Mu = 1.407646882E+16;     {ft^3/s^2}
  Pi = 3.141592654;
  Gnot  = 21.9545;          {mi/hr-s}
  WDot = 31.185;            {lbf/s}

ype
  String6 = string[6];

ar
  DelV1, DelV2, WDry, WLoadUp, WLoadDown, Isp, E1, E2, WI, WF,
  WFuel, Tt, T1, T2, T3, T4, W1, W2, W3, W4,
  A1, A2, A3, AI, AF, AFuel, ASave                         :  Real;
  Value1, Value2, Value3, Value4                           :  String6;
  Code                                                     :  Integer;


rocedure FindDelV (var Value1, Value2:  Real);
const
  Re = 3963.195563;         {mi}
  h1 = 311.0;               {mi}
  h2 = 22236.0;             {mi}
  Mu = 1.407646882E+16;     {ft^3/s^2}
  Theta = 0.497418836;      {rads; 28.5 deg}
ar
  R1, R2, At, V1Circ, V2Circ, V1Per, V2Ap,
  DelVCirc, DelVPlane                              :  Real;
egin
  R1  := Re + h1;              {mi}
  R2  := Re + h2;              {mi}
  At  := (1/2) * (R1 + R2);    {mi}
  V1Circ := SQRT ((Mu/5280) / R1) * (3600/5280);                  {mph}
  V1Per := SQRT ((Mu/5280) * ( 2/R1 - 1/At )) * (3600/5280);
  Value1 := V1Per - V1Circ;
  V2Ap := SQRT ((Mu/5280) * ( 2/R2 - 1/At )) * (3600/5280);
  V2Circ := SQRT ((Mu/5280) / R2) * (3600/5280);
  DelVCirc := V2Circ - V2Ap;
  DelVPlane := 2 * V2Circ * SIN(Theta/2);
  Value2 := SQRT (SQR(DelVCirc) + SQR(DelVPlane));
nd;


EGIN

  textmode(C80);
  textcolor(15);
  textbackground(1);

  WDry := 0.0;
  WLoadUp := 0.0;
```

```
Isp := 0.0;
E1  := 0.0;
E2  := 0.0;
WI  := 0.0;
WF  := 0.0;
WFuel := 0.0;
Delete (Value1, 1, Length(Value1));
Delete (Value2, 1, Length(Value2));
Delete (Value3, 1, Length(Value3));
Delete (Value4, 1, Length(Value4));
Tt := 0.0;

ClrScr;
WriteLn;
WriteLn;
WriteLn ('                OTV Fuel Mass Calculation for Hohmann Transfer');
WriteLn;
WriteLn ('                         Auburn University, Alabama');
WriteLn ('                                  AE 448');
WriteLn;
WriteLn;
Write ('  Enter dry weight of OTV in pounds and hit return key:  ');
ReadLn (Value1);
Val (Value1, WDry, Code);
WriteLn;
Write ('  Enter weight of payload to be delivered to GEO in pounds:  ');
ReadLn (Value2);
Val (Value2, WLoadUp, Code);

WriteLn;
WriteLn ('  Enter weight of payload to be returned from GEO.  (If no payload')

Write ('  is to be returned, enter "0"):  ');
ReadLn (Value4);
Val (Value4, WLoadDown, Code);

WriteLn;
Write ('  Enter specific impulse of engine in seconds:  ');
ReadLn (Value3);
Val (Value3, Isp, Code);

FindDelV (DelV1, DelV2);

E1 := EXP ( DelV1 / (Isp * Gnot));         {Press Cntl-K-D to get out}
E2 := EXP ( DelV2 / (Isp * Gnot));


                         {Main Calculations}
              {For All-Propulsive and Aerobraked Missions}

{Dry OTV + Down Payload in LEO -- OTV in Elliptical Transfer
Orbit at Perigee}
  WF := (WDry + WLoadDown) * E1;
  W4 := WF - WDry - WLoadDown;
  T4 := (WF - WDry - WLoadDown) / WDot;
```

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```
{OTV in ETO at Apogee -- OTV in GEO w/ Down Payload}
  WI := WF;
  WF := WI • E2;
  W3 := WF - WI;
  T3 := (WF - WI) / WDot;

  AF := (WDry + WLoadDown) * E2;
  A3 := AF - WDry - WLoadDown;

{OTV in GEO w/ Down Payload -- OTV in GEO w/ Up Payload}
  WI := WF;
  WF := WI - WLoadDown + WLoadUp;

  AI := AF;
  AF := AI - WLoadDown + WLoadUp;

{OTV in GEO w/ Up Payload -- OTV in ETO at Apogee}
  WI := WF;
  WF := WI * E2;
  W2 := WF - WI;
  T2 := (WF - WI) / WDot;

  AI := AF;
  AF := AI * E2;
  A2 := AF - AI;

{OTV in ETO at Perigee -- OTV in LEO w/ Up Payload}
  WI := WF;
  WF := WI • E1;
  W1 := WF - WI;
  T1 := (WF - WI) / WDot;

  AI := AF;
  AF := AI * E1;
  A1 := AF - AI;

{Weight of Fuel}
  WFuel := W1 + W2 + W3 + W4;
  AFuel := A1 + A2 + A3;
  ASave := WFuel - AFuel;

{Time of transfer}
  Tt := 0.5 * ( 2 * Pi * SQRT ((At*At*At) / Mu) ) ;     {sec}
  Tt := Tt / (3600);                                      {hrs}

ClrScr;
WriteLn;
WriteLn;
WriteLn;
WriteLn ('  Dry OTV Weight:                          ', WDry:6:0, ' lbf');
WriteLn ('  Payload to be Delivered:                 ', WLoadUp:6:0,
          ' lbf');
WriteLn ('  Payload to be Returned:                  ', WLoadDown:6:0,
          ' lbf');
```

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WriteLn ('  Specific Impulse of Engine:                        ', Isp:6:0, ' s');
WriteLn ('  Time of transfer:                                  ', Tt:6:0, ' hrs');
WriteLn;

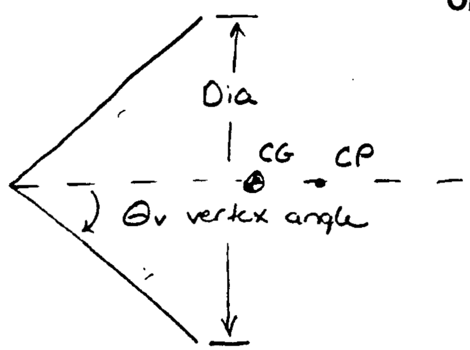
WriteLn ('ALL-PROPULSIVE MISSION:');
WriteLn;
WriteLn ('  Burn times:                                          ', T1:4:0, ',',
          T2:4:0, ',', T3:4:0, ',', T4:4:0, ' s');
WriteLn ('  Weight of Fuel needed for each burn:             ', W1:6:0, ',', W2:6:0,
          ',', W3:6:0, ',', W4:6:0, ' lbf');
Write ('  Total Weight of Fuel required:                    ');
textcolor(1);
textbackground(15);
WriteLn (WFuel:6:0, ' lbf');
textcolor(15);
textbackground(1);

WriteLn;
WriteLn ('AEROBRAKED MISSION:');
WriteLn;
WriteLn ('  Weight of Fuel needed for each burn:             ', A1:6:0, ',', A2:6:0,
         ',', A3:6:0, ' lbf');
Write ('  Total Weight of Fuel required:                    ');
textcolor(1);
textbackground(15);
WriteLn (AFuel:6:0, ' lbf');
textcolor(15);
textbackground(1);

ASave := WFuel - AFuel;
WriteLn ('  Weight of Fuel Saved using Aerobrake:            ', ASave:6:0, ' lbf')
END.
```

Sample Calculation of Center of Pressure

conical lifting brake:



assume $\theta_v = 70°$

assume $\beta = 11 \frac{kg}{m^2}$ (low ballistic coeff ∴ high drag)

$$\beta = \frac{W}{C_D A} = \frac{W}{C_D \frac{\pi}{4} D^2}$$

$$\Rightarrow D = \sqrt{\frac{4W}{C_D \pi \beta}}$$

find C_D from

$$C_D = 2 \sin^2 \theta_v = 2 \sin^2 (70°) = 1.766$$

two scenarios:

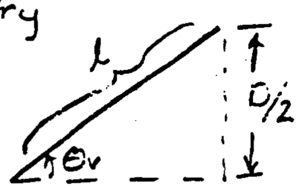
$$W_1 = W_{dry} = 6000 \text{ lbf}$$

$$W_2 = W_{dry} + W_{payload} = (6000 + 5000) \text{ lbf} = 11000 \text{ lbf}$$

$$D_1 = \sqrt{\frac{4(6000 \text{ lbf})\left(\frac{1 \text{ kg}}{2.21 \text{ lbf}}\right)}{(1.766)\pi\left(11 \frac{kg}{m^2}\right)}} = 13.340 \text{ m} = 43.766 \text{ ft}$$

$$D_2 = \sqrt{\frac{4(11000 \text{ lbf})\left(\frac{1}{2.21}\right)}{(1.766)\pi\left(11 \frac{kg}{m^2}\right)}} = 18.062 \text{ m} = 59.259 \text{ ft}$$

from geometry



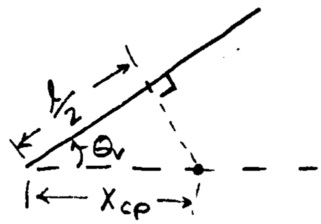
$$\sin \theta_v = \frac{D/2}{l} \Rightarrow l = \frac{D}{2 \sin \theta_v}$$

$$l_1 = \frac{13.340 \text{ m}}{2 \sin 70°} = 7.098 \text{ m}$$

$$l_2 = \frac{18.062 \text{ m}}{2 \sin 70°} = 9.611 \text{ m}$$

cont.

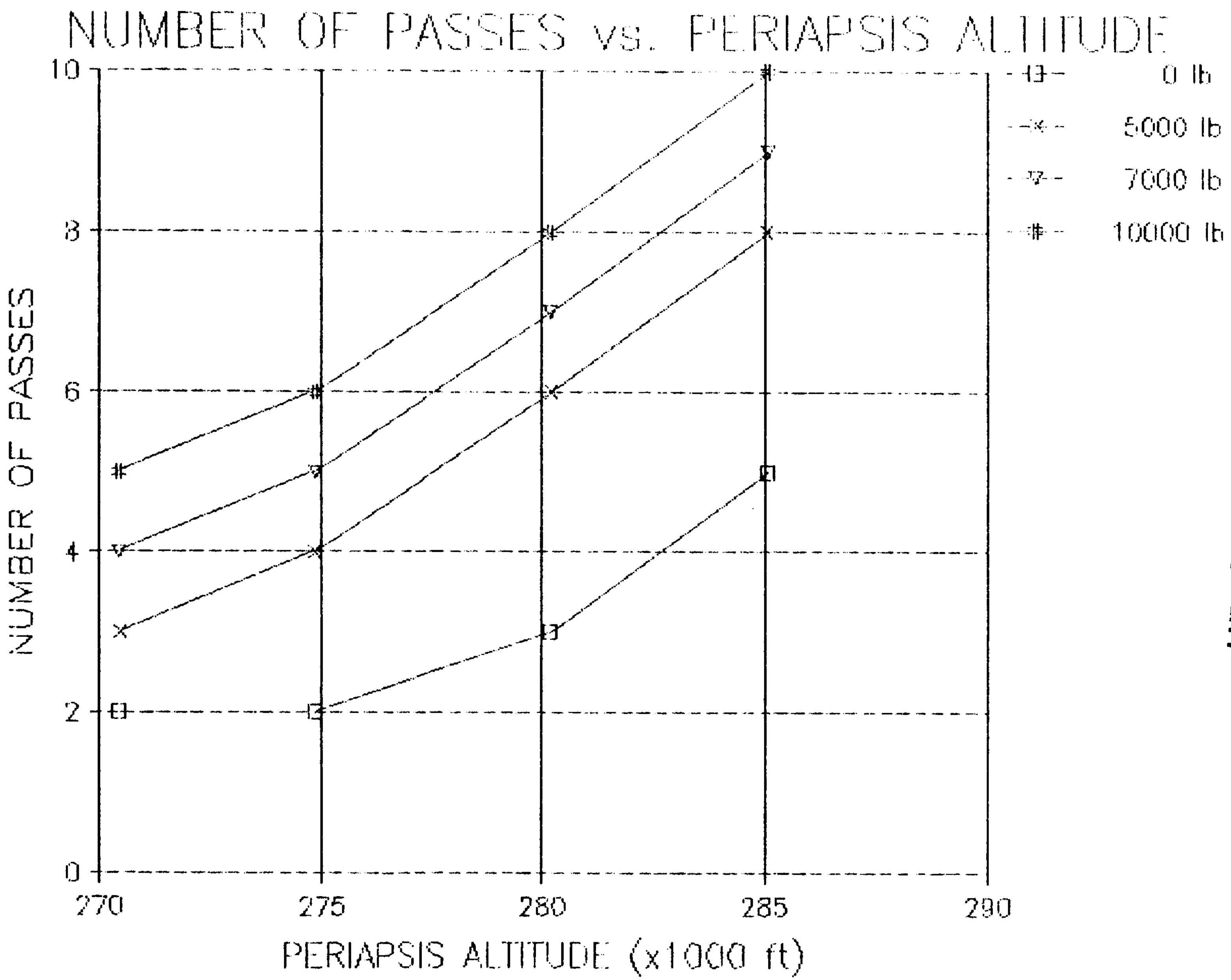
also from geometry



$$\cos\Theta_v = \frac{\ell/2}{X_{cp}} \Rightarrow X_{cp} = \frac{\ell}{2\cos\Theta_v}$$

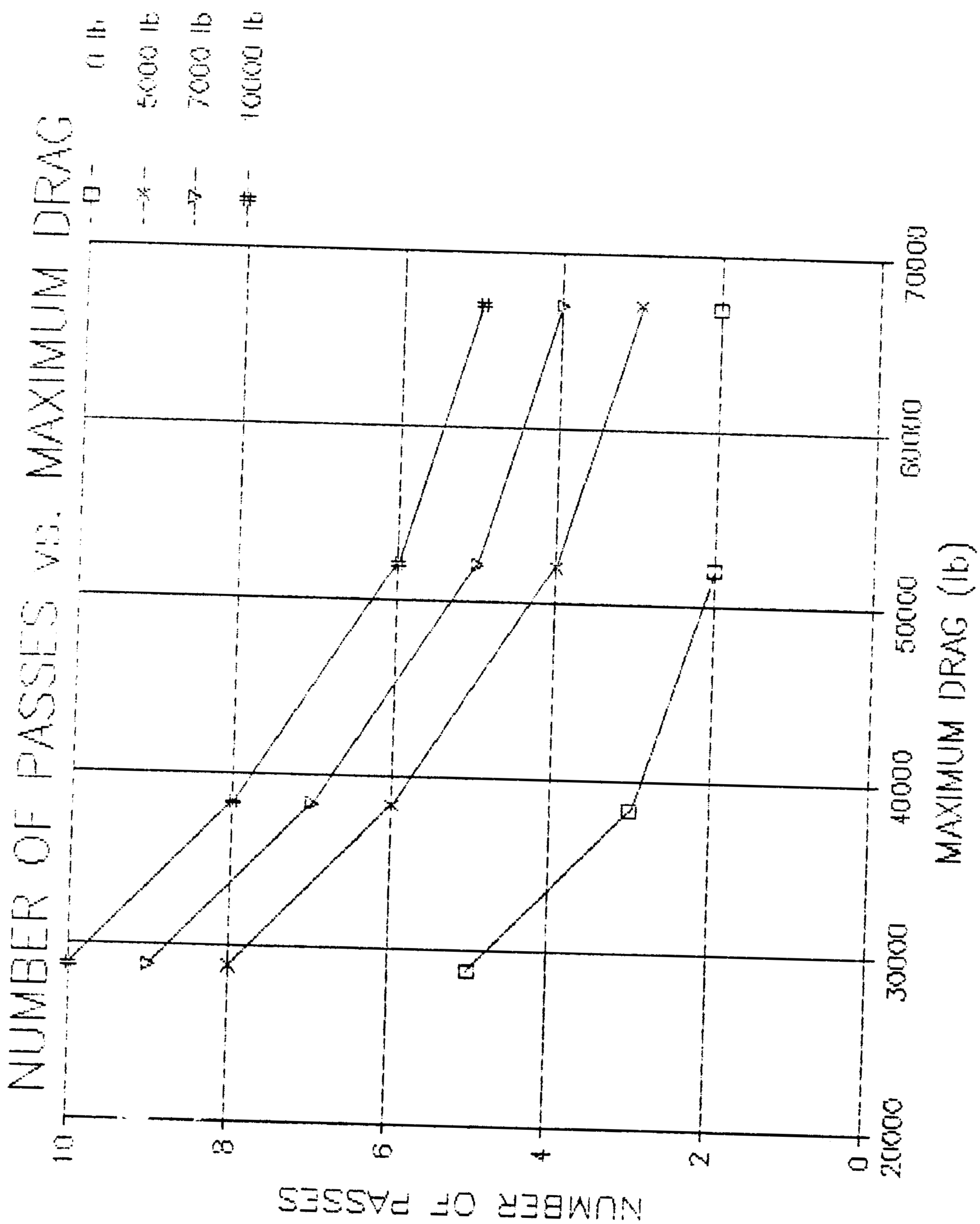
$$X_{cp_1} = \frac{7.098\ m}{2\cos 70^\circ} = 10.377\ m = 32.065\ ft$$

$$X_{cp_2} = \frac{9.611\ m}{2\cos 70^\circ} = 14.050\ m = 43.415\ ft$$

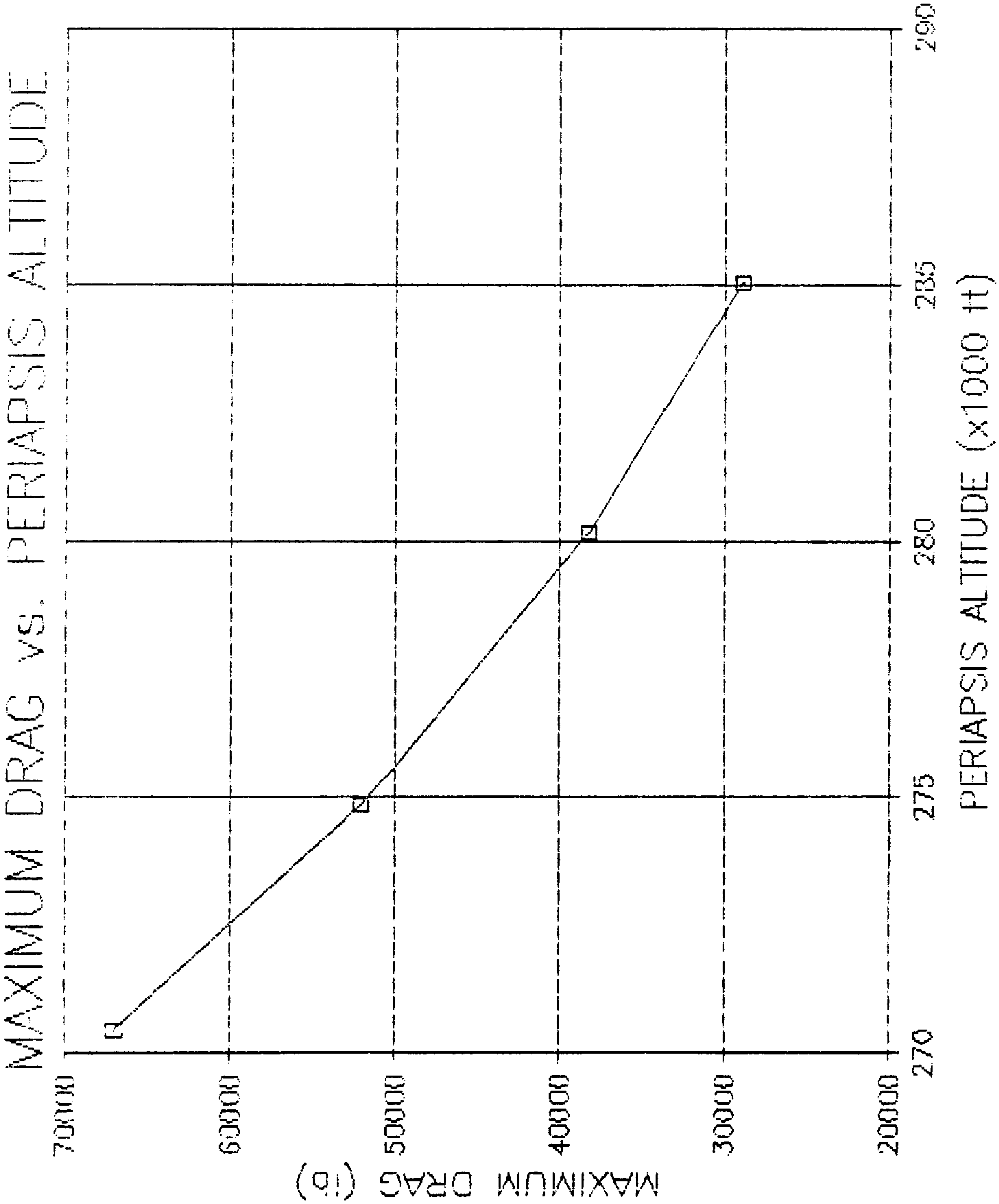


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THIS PROGRAM ANALYZES AN AEROBRAKE MANEUVER THROUGH THE
EARTH'S ATMOSPHERE; UNITS ARE ENGLISH UNITS

   REAL MU,MASS
   OPEN(UNIT=7,FILE='EBRAKE.DAT',STATUS='OLD')
   PRINT*,'INPUT THE PERIAPSIS ALTITUDE IN ft '
   READ(6,*) PERP
DEFINE THE PERIAPSIS FROM THE CENTER OF EARTH
   RE = 2.09256725?E+07
   PERIAP = PERP + RE
THE INITIAL SEMI-MAJOR AXIS IN ft IS (APO AT GEO)
   AINIT = (PERIAP + (22236.0 * 5280.0 + RE)) / 2.0
THE DESIRED APOAPSIS DISTANCE IN ft IS (APO AT LEO)
   APOAFF = 311.0 * 5280.0 + RE
   MU = 1.407646882E+16
   PI = 3.14159?654
CALCULATE THE PARAMETERS OF THE ELLIPTIC ORBIT ABOUT EARTH
USING THE PERIAPSIS FROM INPUT.  THIS ORBIT IS ACTUALLY ONLY
HALF AN ORBIT, MAKING THE JOURNEY FROM APOAPSIS TO AEROBRAKING
PERIAPSIS
   PERIOD = 0.0
   APOAP = 0.0
   CALL PARAMS(PERIAP,APOAP,AINIT,PERIOD)
   PERDHR = PERIOD/3600.
DETERMINE THE TIME FOR THE HALF-ORBIT FROM THE APOAPSIS TO
THE PERIAPSIS OF AEROBRAKING
   TIME = .5*PERIOD
OBTAIN THE INPUTS FOR THE AEROBRAKING PROCESS
   PRINT*,'INPUT THE ATMOSPHERIC DENSITY FOR THE ALTITUDE '
   PRINT*,'SPECIFIED (lbm/ft^3) '
   READ(6,*) RHO
   PRINT*,'INPUT THE RETURNING WEIGHT OF THE SPACE VEHICLE (lbf) '
   READ(6,*) WEIGHT
   MASS = WEIGHT
THE HALF-ANGLE OF THE CONICAL AEROBRAKE (deg)
   THETA = 70.0
   PRINT*,'INPUT THE DIAMETER OF THE AEROBRAKE (ft) '
   READ(6,*) DIAM
DETERMINE THE AREA OF THE CONICAL AEROBRAKE
   THETAR = THETA*PI/180.
   PART = 1./(TAN(THETAR)**2)
   AREA = PI*(DIAM/2.)**2*SQRT(1.+PART)
DETERMINE THE DRAG COEFFICIENT OF THE AEROBRAKE
BASED ON NEWTONIAN METHODS
   CD = 2.*SIN(THETAR)**2
THE PERIAPSIS, APOAPSIS, SEMI-MAJOR AXIS, AND AEROBRAKE DIAMETER
ARE IN ft.  THE AREA IS IN ft^2 AND THE DENSITY IN lbm/ft^3
   TIMTTL = TIME
   TIMTLH = TIMTTL/3600.
   WRITE(7,*)'                AEROBRAKE ANALYSIS   '
   WRITE(7,*)'   '
   WRITE(7,*)'   '
   WRITE(7,*)'   '
   WRITE(7,*)'  HALF-ANGLE FOR CONICAL AEROBRAKE (deg): ',THETA
   WRITE(7,*)'  DIAMETER OF THE AEROBRAKE (ft): ',DIAM
   WRITE(7,*)'  SURFACE AREA OF THE AEROBRAKE (ft^2): ',AREA
   WRITE(7,*)'  RETURN WEIGHT OF SPACE VEHICLE (lbm): ',MASS
   WRITE(7,*)'   '
   WRITE(7,*)'   '
   WRITE(7,*)'  ATMOSPHERIC CONDITIONS: '
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      WRITE(7,*)'   '
      WRITE(7,*)'  INITIAL ORBITAL PARAMETERS:'
      WRITE(7,*)'   '
      WRITE(7,130)
      WRITE(7,*)'   '
      WRITE(7,140) PERIAP,APOAP,AINIT,PERDHR
      WRITE(7,*)'   '
      WRITE(7,*)'   '
      WRITE(7,*)'  APPROX TIME (hr) SPENT IN TRANSFER ORBIT: ',TIMTFR
      WRITE(7,*)'   '
      WRITE(7,*)'   '
      WRITE(7,*)'  AEROBRAKE PROCEDURE:'
      WRITE(7,*)'   '
      WRITE(7,100)
      WRITE(7,110)
      WRITE(7,*)'   '
C SET INITIAL CONDITIONS FOR VARIABLES PRIOR TO DO-LOOP
      SMAJ = AINIT
      X = 0.0
      Y = 0.0
      SEG = 0.0
      PHI = 0.0
      ASECTR = 0.0

      DO 50 I = 1,10
      ECCNTY = (APOAP-PERIAP)/(APOAP+PERIAP)
      SMIN = SQRT(SMAJ**2*(1.-ECCNTY**2))

C CALCULATE THE INTERSECTION POINTS OF THE ELLIPTIC ORBIT
C AND THE ATMOSPHERE
      CALL NTRSEC(SMAJ,SMIN,ECCNTY,X,Y)

C CALCULATE THE LENGTH OF SEGMENT OF THE ELLIPTIC ORBIT
C ENCLOSED BY THE ATMOSPHERE
      CALL SEGMNT(X,Y,SMAJ,ECCNTY,SEG,PHI,ASECTR)

C DETERMINE THE VELOCITY OF THE SPACECRAFT AT PERIAPSIS
      VELCTY = SQRT(MU*(2./PERIAP-1./SMAJ))
C THE SEMI-MAJOR AXIS IS THE "OLD" SEMI-MAJOR AXIS
C THE SEMI-MAJOR AXIS IS THE "OLD" SEMI-MAJOR AXIS

C CALCULATE THE DRAG ON THE VEHICLE DURING THE AEROBRAKING PROCESS
C UNITS ARE (lbf)
      DRAG = .5*CD*RHO*VELCTY**2*AREA/32.174

C DETERMINE THE TIME (IN MINUTES) OF THE AEROBRAKE PASSAGE
      TIME = 2.*ASECTR*SQRT(SMAJ/MU)/SMIN
      TIME = TIME/60.

C DETERMINE THE NEW SEMI-MAJOR AXIS
      ENRGY1 = -MU/(2.*SMAJ)
      SMAJ = -MU/(-2.*DRAG*SEG/MASS+2.*ENRGY1)

C DETERMINE THE PARAMETERS OF THE NEW ELLIPTIC ORBIT
      CALL PARAMS(PERIAP,APOAP,SMAJ,PERIOD)

C PERIOD OF THE ORBIT IS IN HOURS
      PERDHR = PERIOD/3600.
      TIMTTL = TIMTTL + PERIOD

C CHECK IF THE APOAPSIS IS INSIDE THE ATMOSPHERE
      IF(APOAP.LE.21532872.57) GO TO 80

   45 WRITE(7,120) I,PERIAP,APOAP,SMAJ,DRAG,TIME
C CHECK IF THE APOAPSIS IS LESS THAN THE DESIRED APOAPSIS
```

```
      TIMTTL = TIMTTL+PERIOD
   50 CONTINUE

      IF(APOAP.GT.APOAPF) GO TO 85

C   DETERMINE THE TIME TO TRAVEL THE HALF ORBIT FROM THE AEROBRAKING
C   PERIAPSIS TO THE APOAPSIS.
   60 TIMEPA = .5*PERIOD

C   A DELTA-V BURN WILL BE PERFORMED AT THE APOAPSIS TO RAISE THE
C   PERIAPSIS TO LEO.  DETERMINE THE PERIOD OF THE NEW ORBIT, AND
C   THE TIME TO TRAVEL FROM THE APOAPSIS TO THE PERIAPSIS.
      SMAJAP = .5*((311.0*5280.0+RE)+APOAP)
      PERDAP = 2.*PI*SQRT(SMAJAP**3/MU)
      TIMEAP = .5*PERDAP
      IF(APOAP .EQ. APOAPF) GOTO 75
      GO TO 70
C  BECAUSE THE FINAL APOAPSIS FROM AEROBRAKING IS LESS THAN THE DESIRED
C  APOAPSIS, A SMALL DELTA-V BURN WILL HAVE TO BE APPLIED AT THE
C  PERIAPSIS TO RAISE THE APOAPSIS SO THAT THE FINAL CIRCULAR ORBIT IS
C  OBTAINED.  THE PERIOD OF THIS ORBIT, AND IS DETERMINED
   70 SMAJF = 311.0*5280.0+RE
      WRITE(7,*)'AEROBRAKING COMPLETE; SMALL DELTA V REQUIRED TO'
      WRITE(7,*)'RAISE APOAPSIS TO LEO AND FINE-TUNE ORBIT'
      GOTO 90
   75 WRITE(7,*)'AEROBRAKING COMPLETE; NO ADDITIONAL DELTA V REQUIRED'
      GOTO 90

   80 WRITE(7,*)'AEROBRAKING COMPLETE; SMALL DELTA V REQUIRED TO'
      WRITE(7,*)'LOWER APOAPSIS TO LEO AND FINE-TUNE ORBIT'
      GO TO 90

   85 WRITE(7,*)'FINAL APOAPSIS HAS NOT BEEN REACHED WITHIN 10 PASSES'

  130 FORMAT(3X,'PERIAPSIS (ft)',6X,'APOAPSIS (ft)',6x,
     1 'SEMI-MAJOR AXIS',5X,'PERIOD (hrs)')
  140 FORMAT(2X,F15.5,5X,F15.5,5X,F15.5,5X,F11.5)

  100 FORMAT(4X,'PASS',5X,'PERIAPSIS',7X,'APOAPSIS',7X,
     1 'SEMI-MAJOR',6X,'DRAG',8X,'PASSAGE')
  110 FORMAT(3X,'NUMBER',7X,'(ft)',11x,'(ft)',11x,'AXIS (ft)',
     1 5x,'(lbf)',5x,'TIME (min)')
  120 FORMAT(4X,I3,5X,F13.3,3X,F13.3,3X,F13.3,3X,F10.3,3X,F6.3)

   90 CLOSE (UNIT = 7)
      TIMTTL = TIMTTL+TIMEPA+TIMEAP
      TIMHR = TIMTTL/3600.
      WRITE(7,*)'TOTAL TIME (hrs) is: ',TIMHR
      WRITE(7,*)'  (includes time from apoapsis of initial transfer'
      WRITE(7,*)'   orbit through all aerobrake passes and back to'
      WRITE(7,*)'   final apoapsis at LEO)'
      PRINT*,'AEROBRAKING TIME= ',TIMTTL,' s'
      PRINT*,'PER TO APO= ',TIMEPA,' s'
      PRINT*,'APO TO PER= ',TIMEAP,' s'

      STOP
      END

C************************************************************************
C      END OF PROGRAM
C************************************************************************

      SUBROUTINE PARAMS(RP,RA,A,PERD)
C   THIS SUBROUTINE CALCULATES THE APOAPSIS AND PERIOD OF THE
```

```
THE PERIAPSIS, APOAPSIS, AND SEMI-MAJOR AXIS ARE IN ft
THE PERIOD IS IN sec, THE GRAVITATIONAL PARAMETER IS (ft^3/s^2)

   REAL MU
   RA = 2.*A-RP
   PI = 3.141592654
   MU = 1.407645882E+16
   PERD = 2.*PI*SQRT(A**3/MU)
   RETURN
   END

   SUBROUTINE NTRSEC(A,B,E,X,Y)
THIS SUBROUTINE CALCULATES THE POINTS OF INTERSECTION OF THE
SPACE VEHICLE'S ELLIPTIC ORBIT AND THE ATMOSPHERE'S CIRCULAR
ORBIT, USING THE SEMI-MAJOR AXIS, THE ECCENTRICITY, AND THE
SEMI-MINOR AXIS

THE RADIUS OF THE ATMOSPHERE IS IN ft
   RE = 2.092567257E+07
   RADIUS = 115.0*5280.0+RE
   X1 = 2.*A*E
   X2A = 4.*A**2*E**2
   X2B = 4.*(B**2-RADIUS**2-A**2*E**2)*(1.-B**2/A**2)
   X2 = SQRT(X2A-X2B)
   X3 = 2*(1.-B**2/A**2)
THE INTERSECTION POINTS OF THE ELLIPTIC ORBIT ARE X AND Y
   X = (X1+X2)/X3
   Y = SQRT(RADIUS**2-(X-A*E)**2)
   RETURN
   END

   SUBROUTINE SEGMNT(X,Y,A,E,SEG,PHI,AREA)
THIS SUBROUTINE CALCULATES THE LENGTH OF THE SEGMENT (ft) OF THE
SPACE VEHICLE'S ELLIPTIC ORBIT BOUNDED BY THE ATMOSPHERE
USING THE INTERSECTION POINTS, SEMI-MAJOR AXIS, AND ECCENTRICITY
FROM THE MAIN PROGRAM

   C = 2.*Y
   D = X-A*E
   R = SQRT(D**2+Y**2)
PHI IS THE ANGLE OF THE BOUNDED SEGMENT, AND AREA IS THE AREA
OF THE BOUNDED PORTION OF THE ORBIT
   PHI = 2.*ATAN(C/(2.*D))
   SEG = R*PHI
   AREA = .5*R*SEG
   RETURN
   END
```

ORIGINAL PAGE IS
OF POOR QUALITY

TABLE IV.—Continued

GEOMETRIC ALTITUDE, ENGLISH UNITS

Altitude		Temperature			Pressure			Density	
Z, ft	H, ft	T, °R	t, °F	t, °C	P, mb	P, in. Hg	$\frac{P}{P_0}$	ρ, lb ft^{-3}	$\frac{\rho}{\rho_0}$
230000	227491	394.728	-64.942	-53.857	5.43373 - 2	1.60458 - 3	5.36268 - 5	5.3888 - 6	7.0465 - 5
230500	227980	393.654	-66.016	-54.453	5.30882	1.56769	5.23939	5.2793	6.9033
231000	228469	392.581	-67.089	-55.049	5.18644	1.53156	5.11862	5.1717	6.7626
231500	228958	391.508	-68.162	-55.646	5.06658	1.49616	5.00032	5.0660	6.6244
232000	229447	390.434	-69.236	-56.242	4.94917	1.46149	4.88445	4.9622	6.4887
232500	229936	389.361	-70.309	-56.838	4.83417	1.42753	4.77096	4.8603	6.3554
233000	230425	388.288	-71.382	-57.434	4.72155	1.39427	4.65980	4.7601	6.2245
233500	230914	387.215	-72.455	-58.031	4.61125	1.36170	4.55095	4.6618	6.0959
234000	231403	386.142	-73.528	-58.627	4.50324	1.32981	4.44435	4.5653	5.9697
234500	231892	385.069	-74.601	-59.223	4.39747	1.29857	4.33997	4.4705	5.8457
235000	232381	383.996	-75.674	-59.819	4.29391 - 2	1.26799 - 3	4.23776 - 5	4.3774 - 6	5.7240 - 5
235500	232870	382.923	-76.747	-60.415	4.19251	1.23805	4.13769	4.2860	5.6045
236000	233359	381.850	-77.820	-61.011	4.09324	1.20873	4.03972	4.1963	5.4872
236500	233848	380.777	-78.893	-61.607	3.99606	1.18003	3.94380	4.1082	5.3720
237000	234337	379.704	-79.966	-62.203	3.90092	1.15194	3.84990	4.0217	5.2589
237500	234825	378.632	-81.038	-62.799	3.80778	1.12444	3.75799	3.9368	5.1479
238000	235314	377.559	-82.111	-63.395	3.71663	1.09752	3.66803	3.8535	5.0389
238500	235803	376.486	-83.184	-63.991	3.62741	1.07117	3.57997	3.7717	4.9320
239000	236292	375.414	-84.256	-64.587	3.54009	1.04539	3.49379	3.6914	4.8270
239500	236780	374.341	-85.329	-65.183	3.45463	1.02015	3.40946	3.6126	4.7240
240000	237269	373.269	-86.401	-65.779	3.37101 - 2	9.95458 - 4	3.32693 - 5	3.5353 - 6	4.6229 - 5
240500	237758	372.196	-87.474	-66.374	3.28918	9.71294	3.24617	3.4594	4.5237
241000	238246	371.124	-88.546	-66.970	3.20911	9.47651	3.16715	3.3850	4.4263
241500	238735	370.052	-89.618	-67.566	3.13078	9.24518	3.08984	3.3119	4.3308
242000	239224	368.979	-90.691	-68.162	3.05414	9.01887	3.01420	3.2402	4.2370
242500	239712	367.907	-91.763	-68.757	2.97916	8.79746	2.94020	3.1699	4.1451
243000	240201	366.835	-92.835	-69.353	2.90582	8.58089	2.86782	3.1009	4.0548
243500	240689	365.763	-93.907	-69.948	2.83408	8.36904	2.79702	3.0332	3.9663
244000	241178	364.691	-94.979	-70.544	2.76391	8.16183	2.72777	2.9668	3.8795
244500	241666	363.619	-96.051	-71.140	2.69529	7.95918	2.66004	2.9017	3.7943
245000	242155	362.547	-97.123	-71.735	2.62817 - 2	7.76099 - 4	2.59380 - 5	2.8378 - 6	3.7108 - 5
245500	242643	361.475	-98.195	-72.331	2.56254	7.56718	2.52903	2.7751	3.6288
246000	243132	360.40	-99.27	-72.93	2.4984	7.3777	2.4657	2.714	3.548
246500	243620	359.33	-100.34	-73.52	2.4356	7.1924	2.4038	2.653	3.470
247000	244108	358.26	-101.41	-74.12	2.3743	7.0112	2.3432	2.594	3.392
247500	244597	357.19	-102.48	-74.71	2.3143	6.8341	2.2840	2.536	3.317
248000	245085	356.12	-103.55	-75.31	2.2556	6.6609	2.2261	2.480	3.242
248500	245573	355.04	-104.63	-75.90	2.1983	6.4916	2.1696	2.424	3.169
249000	246062	353.97	-105.70	-76.50	2.1423	6.3262	2.1143	2.369	3.098
249500	246550	352.90	-106.77	-77.09	2.0875	6.1645	2.0602	2.316	3.028
250000	247038	351.83	-107.84	-77.69	2.0340 - 2	6.0065 - 4	2.0074 - 5	2.263 - 6	2.959 - 5
250500	247526	350.76	-108.91	-78.28	1.9817	5.8520	1.9558	2.212	2.892
251000	248015	349.69	-109.98	-78.88	1.9306	5.7011	1.9054	2.161	2.826
251500	248503	348.62	-111.05	-79.47	1.8807	5.5537	1.8561	2.112	2.761
252000	248991	347.54	-112.13	-80.07	1.8319	5.4096	1.8079	2.063	2.698
252500	249479	346.47	-113.20	-80.66	1.7842	5.2688	1.7609	2.016	2.636
253000	249967	345.40	-114.27	-81.26	1.7377	5.1313	1.7149	1.969	2.575
253500	250455	344.33	-115.34	-81.85	1.6922	4.9970	1.6700	1.924	2.516
254000	250943	343.26	-116.41	-82.45	1.6477	4.8658	1.6262	1.879	2.457
254500	251431	342.19	-117.48	-83.05	1.6043	4.7376	1.5834	1.835	2.400
255000	251919	341.12	-118.55	-83.64	1.5620 - 2	4.6125 - 4	1.5415 - 5	1.792 - 6	2.344 - 5
255500	252407	340.05	-119.62	-84.23	1.5206	4.4902	1.5007	1.750	2.289
256000	252895	338.98	-120.69	-84.83	1.4802	4.3709	1.4608	1.709	2.235
256500	253383	337.91	-121.76	-85.42	1.4407	4.2544	1.4219	1.669	2.182
257000	253871	336.83	-122.84	-86.02	1.4022	4.1406	1.3838	1.630	2.131
257500	254359	335.76	-123.91	-86.61	1.3646	4.0295	1.3467	1.591	2.080
258000	254847	334.69	-124.98	-87.21	1.3278	3.9211	1.3105	1.553	2.031
258500	255335	333.62	-126.05	-87.80	1.2920	3.8153	1.2751	1.516	1.982
259000	255822	332.55	-127.12	-88.40	1.2570	3.7119	1.2406	1.480	1.935
259500	256310	331.48	-128.19	-88.99	1.2229	3.6111	1.2069	1.444	1.888
260000	256798	330.41	-129.26	-89.59	1.1895 - 2	3.5127 - 4	1.1740 - 5	1.409 - 6	1.843 - 5
260500	257286	329.34	-130.33	-90.18	1.1570	3.4167	1.1419	1.375	1.798
261000	257774	328.27	-131.40	-90.78	1.1253	3.3230	1.1106	1.342	1.755
261500	258261	327.20	-132.47	-91.37	1.0944	3.2316	1.0800	1.309	1.712
262000	258749	326.13	-133.54	-91.97	1.0642	3.1425	1.0502	1.277	1.670
262500	259237	325.17	-134.50	-92.50	1.0347	3.0555	1.0212	1.246	1.629
263000	259724	325.17	-134.50	-92.50	1.0060	2.9708	9.9287 - 6	1.211	1.584
263500	260212	325.17	-134.50	-92.50	9.7814 - 3	2.8884	9.6535	1.178	1.540
264000	260699	325.17	-134.50	-92.50	9.5103	2.8084	9.3859	1.145	1.497
264500	261187	325.17	-134.50	-92.50	9.2468	2.7306	9.1258	1.113	1.456
265000	261675	325.17	-134.50	-92.50	8.9905 - 3	2.6549 - 4	8.8729 - 6	1.082 - 6	1.415 - 5
265500	262162	325.17	-134.50	-92.50	8.7414	2.5813	8.6271	1.052	1.376
266000	262650	325.17	-134.50	-92.50	8.4992	2.5098	8.3880	1.023	1.338
266500	263137	325.17	-134.50	-92.50	8.2637	2.4403	8.1556	9.948 - 7	1.301
267000	263624	325.17	-134.50	-92.50	8.0347	2.3727	7.9297	9.673	1.265
267500	264112	325.17	-134.50	-92.50	7.8121	2.3069	7.7100	9.405	1.230
268000	264599	325.17	-134.50	-92.50	7.5957	2.2430	7.4964	9.144	1.196
268500	265087	325.17	-134.50	-92.50	7.3853	2.1809	7.2887	8.891	1.163
269000	265574	325.17	-134.50	-92.50	7.1807	2.1205	7.0868	8.645	1.130
269500	266061	325.17	-134.50	-92.50	6.9818	2.0617	6.8905	8.405	1.099

TABLE IV.—Continued

GEOMETRIC ALTITUDE, ENGLISH UNITS

Altitude		Temperature			Pressure			Density	
Z, ft	H, ft	T, °R	t, °F	t, °C	P, mb	P, in. Hg	$\frac{P}{P_0}$	ρ, lb ft^{-3}	$\frac{\rho}{\rho_0}$
270000	266549	325.17	-134.50	-92.50	6.7884 - 3	2.0046 - 4	6.6996 - 6	8.172 - 7	1.069 - 5
270500	267036	325.17	-134.50	-92.50	6.6004	1.9491	6.5141	7.946	1.039
271000	267523	325.17	-134.50	-92.50	6.4176	1.8951	6.3337	7.726	1.010
271500	268010	325.17	-134.50	-92.50	6.2399	1.8426	6.1583	7.512	9.823 - 6
272000	268498	325.17	-134.50	-92.50	6.0671	1.7916	5.9877	7.304	9.551
272500	268985	325.17	-134.50	-92.50	5.8990	1.7420	5.8219	7.102	9.286
273000	269472	325.17	-134.50	-92.50	5.7357	1.6937	5.6607	6.905	9.029
273500	269959	325.17	-134.50	-92.50	5.5769	1.6469	5.5039	6.714	8.779
274000	270446	325.17	-134.50	-92.50	5.4225	1.6013	5.3516	6.528	8.536
274500	270933	325.17	-134.50	-92.50	5.2723	1.5569	5.2034	6.347	8.300
275000	271420	325.17	-134.50	-92.50	5.1264 - 3	1.5138 - 4	5.0593 - 6	6.171 - 7	8.070 - 6
275500	271908	325.17	-134.50	-92.50	4.9844	1.4719	4.9193	6.001	7.847
276000	272395	325.17	-134.50	-92.50	4.8465	1.4312	4.7831	5.835	7.629
276500	272882	325.17	-134.50	-92.50	4.7123	1.3915	4.6507	5.673	7.418
277000	273369	325.17	-134.50	-92.50	4.5819	1.3530	4.5219	5.516	7.213
277500	273856	325.17	-134.50	-92.50	4.4550	1.3156	4.3968	5.363	7.013
278000	274342	325.17	-134.50	-92.50	4.3317	1.2792	4.2751	5.215	6.819
278500	274829	325.17	-134.50	-92.50	4.2118	1.2438	4.1568	5.071	6.630
279000	275316	325.17	-134.50	-92.50	4.0953	1.2093	4.0417	4.930	6.447
279500	275803	325.17	-134.50	-92.50	3.9820	1.1759	3.9299	4.794	6.268
280000	276290	325.17	-134.50	-92.50	3.8718 - 3	1.1433 - 4	3.8211 - 6	4.661 - 7	6.095 - 6
280500	276777	325.17	-134.50	-92.50	3.7646	1.1117	3.7154	4.532	5.926
281000	277264	325.17	-134.50	-92.50	3.6605	1.0809	3.6126	4.407	5.762
281500	277750	325.17	-134.50	-92.50	3.5592	1.0510	3.5126	4.285	5.603
282000	278237	325.17	-134.50	-92.50	3.4607	1.0219	3.4154	4.166	5.448
282500	278724	325.17	-134.50	-92.50	3.3650	9.9367 - 5	3.3210	4.051	5.297
283000	279211	325.17	-134.50	-92.50	3.2719	9.6618	3.2291	3.939	5.151
283500	279697	325.17	-134.50	-92.50	3.1814	9.3945	3.1398	3.830	5.008
284000	280184	325.17	-134.50	-92.50	3.0934	9.1347	3.0529	3.724	4.870
284500	280671	325.17	-134.50	-92.50	3.0078	8.8820	2.9685	3.621	4.735
285000	281157	325.17	-134.50	-92.50	2.9246 - 3	8.6363 - 5	2.8863 - 6	3.521 - 7	4.604 - 6
285500	281644	325.17	-134.50	-92.50	2.8437	8.3974	2.8065	3.423	4.477
286000	282130	325.17	-134.50	-92.50	2.7651	8.1652	2.7289	3.329	4.353
286500	282617	325.17	-134.50	-92.50	2.6886	7.9394	2.6534	3.237	4.232
287000	283103	325.17	-134.50	-92.50	2.6142	7.7198	2.5800	3.147	4.115
287500	283590	325.17	-134.50	-92.50	2.5419	7.5063	2.5087	3.060	4.002
288000	284076	325.17	-134.50	-92.50	2.4716	7.2988	2.4393	2.976	3.891
288500	284563	325.17	-134.50	-92.50	2.4033	7.0970	2.3719	2.893	3.783
289000	285049	325.17	-134.50	-92.50	2.3369	6.9007	2.3063	2.813	3.679
289500	285536	325.17	-134.50	-92.50	2.2722	6.7099	2.2425	2.735	3.577
290000	286022	325.17	-134.50	-92.50	2.2094 - 3	6.5244 - 5	2.1805 - 6	2.660 - 7	3.478 - 6
290500	286509	325.17	-134.50	-92.50	2.1483	6.3441	2.1203	2.586	3.382
291000	286995	325.17	-134.50	-92.50	2.0890	6.1687	2.0616	2.515	3.288
291500	287481	325.17	-134.50	-92.50	2.0312	5.9982	2.0046	2.445	3.198
292000	287967	325.17	-134.50	-92.50	1.9751	5.8324	1.9492	2.378	3.109
292500	288454	325.17	-134.50	-92.50	1.9205	5.6712	1.8954	2.312	3.023
293000	288940	325.17	-134.50	-92.50	1.8674	5.5144	1.8430	2.248	2.940
293500	289426	325.17	-134.50	-92.50	1.8158	5.3620	1.7920	2.186	2.858
294000	289912	325.17	-134.50	-92.50	1.7656	5.2138	1.7425	2.126	2.779
294500	290399	325.17	-134.50	-92.50	1.7168	5.0697	1.6944	2.067	2.703
295000	290885	325.17	-134.50	-92.50	1.6694 - 3	4.9296 - 5	1.6475 - 6	2.010 - 7	2.628 - 6
295500	291371	325.54	-134.13	-92.30	1.6232	4.7934	1.6020	1.952	2.552
296000	291857	326.36	-133.31	-91.84	1.5785	4.6613	1.5579	1.893	2.476
296500	292343	327.17	-132.50	-91.39	1.5351	4.5331	1.5150	1.837	2.402
297000	292829	327.99	-131.68	-90.93	1.4930	4.4088	1.4735	1.782	2.330
297500	293315	328.81	-130.86	-90.48	1.4522	4.2882	1.4332	1.729	2.261
298000	293801	329.63	-130.04	-90.02	1.4125	4.1712	1.3941	1.677	2.193
298500	294287	330.45	-129.22	-89.57	1.3741	4.0576	1.3561	1.628	2.128
299000	294773	331.27	-128.40	-89.11	1.3368	3.9475	1.3193	1.580	2.065
299500	295259	332.09	-127.58	-88.66	1.3006	3.8405	1.2835	1.533	2.004
300000	295745	332.90	-126.77	-88.20	1.2654 - 3	3.7368 - 5	1.2489 - 6	1.488 - 7	1.946 - 6

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Calculation of Thickness and Mass of Propellant Tanks

Determined Tank Pressures:

LH_2: 38 psi at tank bottom (maximum)

LO_2: 42 psi at tank side
48.5 psi at tank bottom

σ_y for Aluminum 2219 T87: 67,000 psi (yield strength)

1. Considering the LH_2 tank:

for a spherical tank: $\sigma_t = \frac{PR}{2t}$ (Ref.)

using a factor of safety of 1.5:

$$\frac{67{,}000\,\text{psi}}{1.5} = \frac{(38\,\text{psi})(86.22\,\text{in})}{2t}$$

$$t = .0367\ \text{in}$$

mass of the LH_2 tank: $m = \rho V$

$$V = St$$

$$S = 4\pi R^2 = 4\pi(86.22\,\text{in})^2 = 93{,}417\,\text{in}^2$$

$$V = St = (93{,}417\,\text{in}^2)(.0367\,\text{in}) = 3{,}428.4\,\text{in}^3$$

$$m = \rho V = (.1\,\text{lbm/in}^3)(3{,}428.4\,\text{in}^3) = 342.8\,\text{lbm}$$

$$m = 342.8\ \text{lbm}$$

2. Considering the LO_2 tank:

for an oblate spheroid: $\sigma_t = \frac{PR}{t}\left[1 - \left(\frac{R^2}{2h^2}\right)\right]$ side of tank

$\sigma_t = \frac{PR^2}{2th}$ bottom of tank (Ref.)

finding thickness from each equation to determine value of greatest thickness required, and using a factor of safety of 1.5:

$$\frac{67{,}000\,\text{psi}}{1.5} = \frac{(42\,\text{psi})(77.86\,\text{in})}{t}\left[1 - \left(\frac{(77.86\,\text{in})^2}{2(38.93\,\text{in})^2}\right)\right]$$

$$t = .0586\ \text{in (at tank side)}$$

$$\frac{67{,}000\,\text{psi}}{1.5} = \frac{(48.5\,\text{psi})(77.86\,\text{in})^2}{2(38.93\,\text{in})t}$$

$$t = .0845\ \text{in (at tank bottom)}$$

$\therefore t = .0845$ in

mass of the LO_2 tank: $m = \rho \forall$

$\forall = St$

$S = 2\pi a^2 + \pi \frac{b^2}{\epsilon} \log_e \frac{1+\epsilon}{1-\epsilon}$

solve for ϵ, "eccentricity": $e = \sqrt{-\left(\frac{b}{a}\right)^2 + 1}$

$e = \sqrt{-\left(\frac{38.93}{77.86}\right)^2 + 1}$

$e = .8660$

$\therefore S = 2\pi(77.86in)^2 + \pi \frac{(38.93in)^2}{.8660} \ln \frac{1+.8660}{1-.8660} = 52,570 in^2$

$\forall = St = (52,570 in^2)(.0845 in) = 4,442.15 in^3$

$m = \rho \forall = (.1\ lbm/in^3)(4,442.15 in^3) = 444.2\ lbm$

$m = 444.2\ lbm$

3. Total Propellant Tank Mass:

$m_{LH_{2_{tank}}} + m_{LO_{2_{tank}}} = 342.8\ lbm + 444.2\ lbm$

$m_{Total} = 787.0\ lbm$

Dimension and Volume Calculations for Propellant Tanks and Calculation of Propellant Mass

Hydrogen tank was sized according to the maximum diameter which would fit in the Shuttle bay, i.e. a diameter of 15 ft. a clearance of just over 0.3 ft on all sides was allowed:

$$D_{H_2 tank} = 86.22 in$$

$$\forall_{H_2 tank} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{86.22 in}{2}\right)^3$$

$$\forall_{H_2 tank} = 1{,}552.6 ft^3$$

Volume of oxygen tank was found from the mass of H_2 used and a 6:1 oxidizer to fuel ratio:

$$M_{H_2} = \rho_{H_2}\forall_{H_2} = (70.8 \tfrac{Kg}{m^3})(1{,}552.6 ft^3)(.02832 \tfrac{m^3}{ft^3})$$

$$M_{H_2} = 3112.70\ Kg \left(\frac{2.20462\ lbm}{Kg}\right) = 6{,}862.3\ lbm$$

$$\frac{M_o}{M_f} = r = 6$$

$$\frac{M_o}{6{,}862.3 lbm} = 6, \quad M_{O_2} = 41{,}038.2\ lbm$$

$$\forall_{O_2 tank} = \frac{M_{O_2}}{\rho_{O_2}} = \frac{41{,}038.2\ lbm}{1{,}149 \frac{Kg}{m^3}}\left(\frac{1 Kg}{2.20462\ lbm}\right)$$

$$\forall_{O_2 tank} = 16.2 m^3 \left(\frac{1}{.02832 \frac{m^3}{ft^3}}\right) = 572.1 ft^3$$

Taking a 2:1 ratio of semi-major axis to semi-minor axis to give minimum weight requirement for an elliptically-shaped tank, the dimensions of the LO_2 tank are found:

$$\forall_{O_2 tank} = \frac{4}{3}\pi a^2 b = \frac{4}{3}\pi R^2\left(\frac{R}{2}\right)$$

$$572.1 ft^3 = \frac{4}{3}\pi \frac{R^3}{2}$$

$$R = a = 6.488 ft$$

$$\text{and } b = 3.244 ft$$

And total mass of the propellants:

$$M_{O_2} + M_{H_2} = 41{,}038.2\ lbm + 6{,}862.3\ lbm$$

$$M_{Total} = 47{,}900\ lbm$$

Calculating the approximate weight of the avionics section. Source (2:182)

① Fuel Cell Power Plant:

Shuttle power plant (from which DARVES was scaled down, weighs 200 lb

but our lightweight fuel cell is 4 lb/kW vs. Shuttle's 8 lb/kW.

Power Plant supplies 12 kW

(12 kW)(8 lb/kW) = 96 lb power plant contribution to power plant itself. (for Shuttle)

200 − 96 lb = 104 lb ; Shuttle structure weight

DARVES = 3/4 Shuttle

Darves fuel cell power plant structure weight = 3/4(104 lb) = 78 lb

∴ Total Power plant weight = 78 lb + (4 lb/kW)(12 kW)

Total Power Plant Weight = 126 lb

Avg. Size of Ni/Cd battery ≈ 40 lbs

② Communications

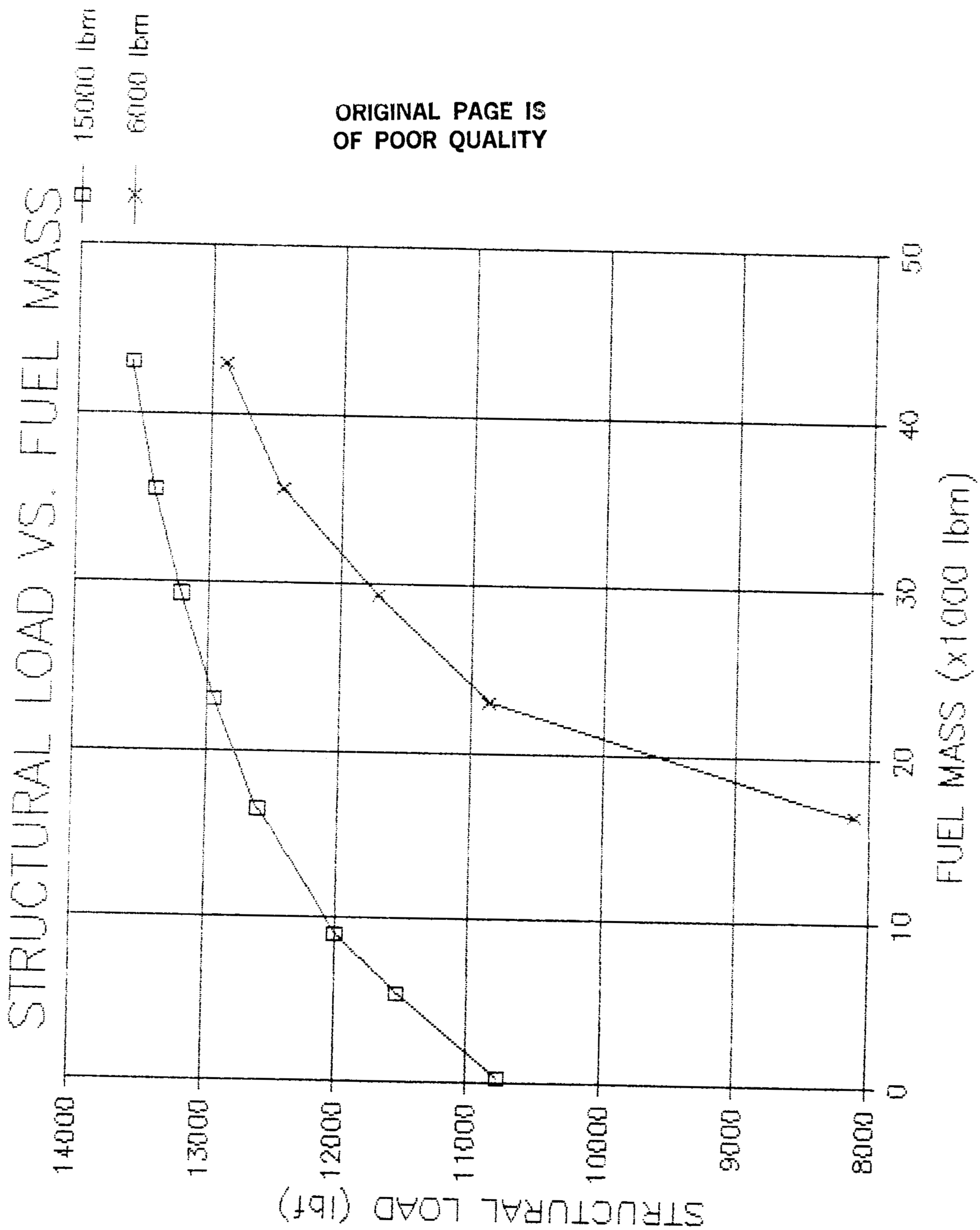
Using State of the art Technology = 326 lbs

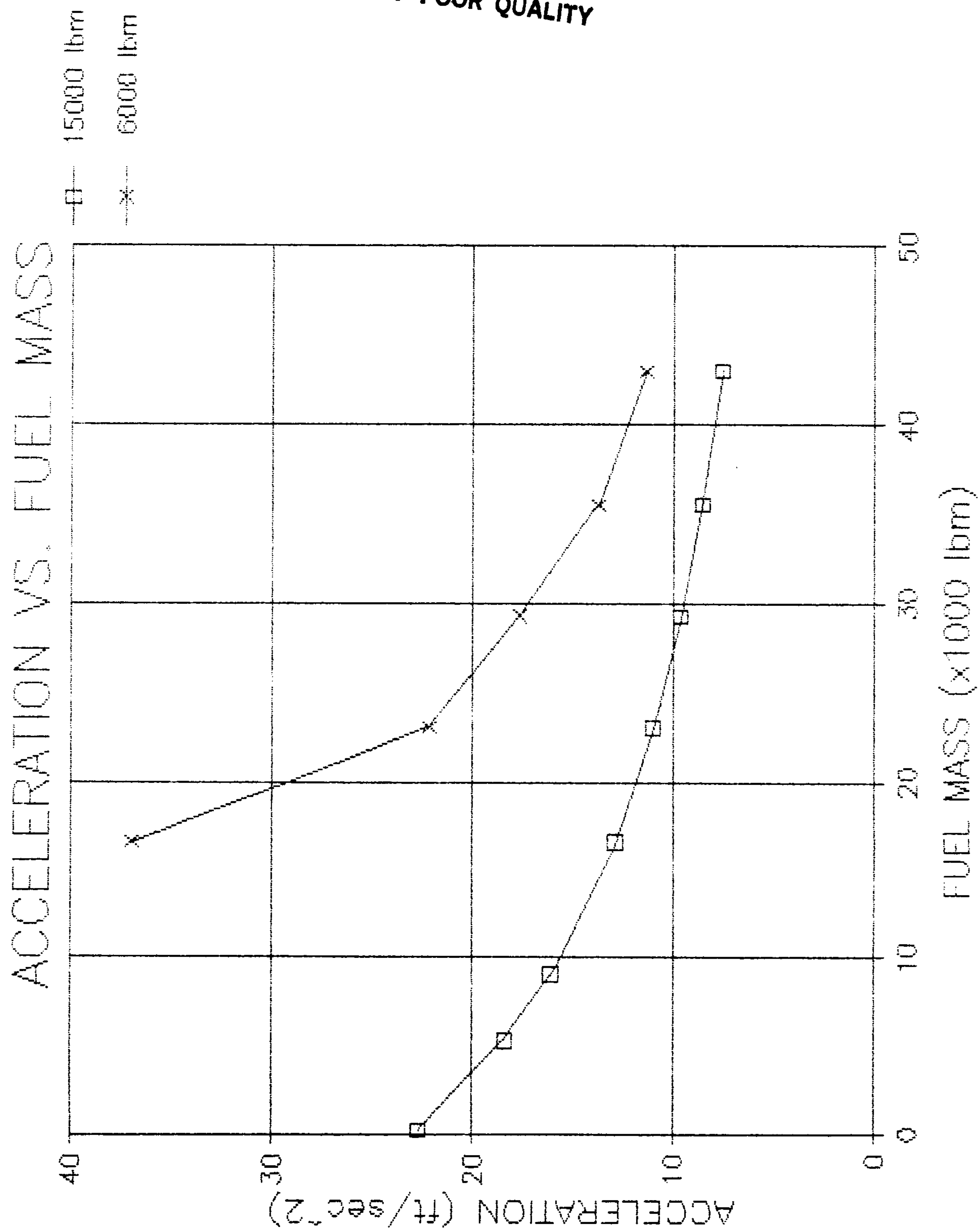
③ Guidance and Control

Bulk of Weight will be here ≈ 550 lbs

TOTAL WEIGHT = 1042 lbs

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ACCELERATION VS. FUEL MASS
15000 lbm
6000 lbm
ACCELERATION (ft/sec^2)
0
10
20
30
40
FUEL MASS (x1000 lbm)
0
10
20
30
40
50

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Numerical Integration to calculate Acceleration
15000 lb Payload

dV/dT	TIME (SEC)		MFORCE O2	MFORCE H2	force PAYL	TOTAL FORCE
7.546875	*CONDITIONS		*8638.359	*1439.765	3515.625	13593.75
7.578975	*DRY WT.=	7140	*8617.702	*1436.322	3532.839	13586.86
7.621145	*PAYLOAD	15000	*8596.841	*1432.846	3550.223	13579.91
7.658031	*FUEL LOAD	43000	*8575.774	*1429.335	3567.778	13572.88
7.696392	*THRUST=	15000	*8554.498	*1425.789	3585.508	13565.79
[illegible]	*		*8533.007	*1422.200	3603.416	13558.63
7.774159	*		*8511.304	*1418.591	3621.503	13551.39
7.813077	*		*8489.381	*1414.937	3639.772	13544.09
7.851994	*		*8467.235	*1411.246	3658.227	13536.70
[illegible]	*		*8444.863	*1407.518	3676.870	13529.25
7.937444	*		*8422.262	*1403.751	3695.703	13521.71
7.974291	*		*8399.429	*1399.946	3714.731	13514.10
8.015559	*		*8376.259	*1396.101	3733.956	13506.41
[illegible]	*		*8353.049	*1392.216	3753.381	13498.64
8.093372	*		*8329.496	*1388.291	3773.009	13490.77
8.14[illegible]	*		*8305.694	*1384.324	3792.843	13482.86
8.184777	*		*8281.641	*1380.316	3812.887	13474.84
[illegible]	*		*8257.333	*1376.264	3833.144	13466.74
8.272471	*		*8232.765	*1372.170	3853.617	13458.55
8.316857	*		*8207.933	*1368.031	3874.310	13450.27
8.361734	*		*8182.833	*1363.848	3895.227	13441.90
8.407142	*		*8157.460	*1359.620	3916.370	13433.45
8.453034	*		*8131.811	*1355.345	3937.745	13424.90
8.499410	*		*8105.880	*1351.024	3959.354	13416.25
8.546713	*		*8079.662	*1346.654	3981.201	13407.51
8.593732	*		*8053.154	*1342.236	4003.291	13398.68
8.641681	*		*8026.350	*1337.769	4025.628	13389.74
8.690112	*		*7999.246	*1333.252	4048.215	13380.71
8.739202	*		*7971.835	*1328.684	4071.057	13371.57
8.788793	*		*7944.113	*1324.064	4094.158	13362.33
8.838747	*		*7916.075	*1319.391	4117.523	13352.99
8.889480	*		*7887.715	*1314.665	4141.156	13343.53
8.941000	*		*7859.028	*1309.884	4165.062	13333.97
8.993713	*		*7830.007	*1305.047	4189.245	13324.30
9.045475	*		*7800.648	*1300.154	4213.711	13314.51
9.098572	*		*7770.943	*1295.204	4238.465	13304.61
9.152333	*		*7740.888	*1290.195	4263.511	13294.59
9.206742	*		*7710.475	*1285.126	4288.855	13284.45
9.261798	*		*7679.699	*1279.997	4314.502	13274.19
9.317516	*		*7648.552	*1274.806	4340.457	13263.81
9.373908	*		*7617.028	*1269.553	4366.727	13253.30
9.430987	*		*7585.120	*1264.235	4393.317	13242.67
9.488745	*		*7552.821	*1258.852	4420.232	13231.90
9.547250	*		*7520.124	*1253.403	4447.479	13221.06
9.606470	*		*7487.022	*1247.886	4475.064	13209.97
9.666425	*		*7453.506	*1242.301	4502.994	13198.80
9.727136	*		*7419.569	*1236.645	4531.274	13187.49
9.788610	*		*7385.204	*1230.910	4559.912	13176.03
9.850870	*		*7350.401	*1225.117	4588.914	13164.43
9.913925	*		*7315.153	*1219.243	4618.288	13152.68
9.977792	*		*7279.450	*1213.293	4648.040	13140.78
10.04248	*		*7243.284	*1207.266	4678.177	13128.72
10.10802	*		*7206.647	*1201.160	4708.708	13116.51
10.17447	*		*7169.528	*1194.974	4739.64[illegible]	13104.14

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10.37877	*7055.186	*1175.918	4834.924	13066.03
[illegible]	*7016.044	*1169.374	4867.343	13052.98
[illegible]	*6976.370	*1162.782	4900.604	13039.75
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10.66484	*6895.382	*1149.285	4968.093	13012.76
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10.81374	*6812.133	*1135.411	5037.467	12985.01
10.89779	*6769.631	*1128.330	5072.886	12970.84
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13.10041	*5533.869	*922.3793	6102.679	12558.92
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13.44149	*5343.203	*890.6034	6261.566	12495.37
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13.67892	*5210.479	*868.4841	6372.168	12451.13
13.80090	*5142.344	*857.1287	6428.947	12428.42
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[illegible]	*3859.575	*643.3457	7497.913	12000.83

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[illegible] *	*3745.090	*627.5992	7576.650	11969.33
[illegible] *	*3668.579	*611.5183	7657.050	11937.17
[illegible] *	*3570.037	*595.0923	7739.192	11904.32
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[illegible] *	*3366.433	*561.1601	7906.361	11836.45
[illegible] *	*3261.246	*543.6293	7996.817	11801.37
[illegible] *	*3153.701	*525.7067	8086.137	11765.54
[illegible] *	*3043.718	*507.3772	8177.789	11728.88
17.75624 *	*2931.213	*488.6275	8271.541	11691.38
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[illegible] *	*2698.284	*449.8081	8465.648	11613.74
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[illegible] *	*2454.157	*409.1229	8669.053	11532.36
[illegible] *	*2327.644	*388.0361	8774.512	11490.17
[illegible] *	*2198.013	*366.4342	8882.537	11446.98
19.30550 *	*2065.151	*344.2917	8993.255	11402.69
19.54914 *	*1929.934	*321.5903	9106.767	11357.29
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20.05547 *	*1645.919	*274.4236	9342.612	11262.95
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20.58817 *	*1347.854	*224.7490	9590.997	11163.60
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[illegible] *	*869.8074	*145.0789	9989.366	11004.25
[illegible] *	*701.5095	*117.0308	10129.61	10948.15
[illegible] *	*528.4186	*88.10392	10273.85	10890.45
[illegible] *	*350.3270	*58.50364	10422.26	10831.09
[illegible] *	*167.0140	*27.95331	10575.02	10769.99
[illegible] *	*-21.7508	*-3.50589	10732.32	10707.06

Numerical Integration to Calculate Acceleration

6000 lb Payload

TIME (SEC)			MFORCE DD	MFORCE HD	force PAYL	TOTAL FORCE
[illegible]	*IGNITION		*9268.953	*1544.260	2093.023	12906.83
[illegible]	*INIT WT.=	7140	*9242.740	*1540.491	2100.313	12891.54
[illegible]	[illegible]	[illegible]	*9216.142	*1536.059	2123.828	12876.02
[illegible]	*TOTAL LOAD	71000	*9189.149	*1531.560	2139.577	12860.28
11.76814	*THRUST=	15000	*9161.752	*1526.994	2155.554	12844.30
[illegible]			*9133.944	*1522.360	2171.775	12828.07
[illegible]			*9105.713	*1517.655	2188.242	12811.61
[illegible]			*9077.050	*1512.878	2204.960	12794.89
11.92437			*9047.949	*1508.028	2221.936	12777.91
[illegible]			*9018.394	*1503.103	2239.176	12760.67
[illegible]			*8988.378	*1498.100	2256.685	12743.16
[illegible]			*8957.880	*1493.019	2274.470	12725.37
[illegible]			*8926.913	*1487.857	2292.538	12707.30
[illegible]			*8895.447	*1482.612	2310.894	12688.95
[illegible]			*8863.465	*1477.283	2329.548	12670.29
[illegible]			*8830.966	*1471.866	2348.505	12651.33
[illegible]			*8797.934	*1466.361	2367.773	12632.06
[illegible]			*8764.355	*1460.765	2387.359	12612.40
[illegible]			*8730.216	*1455.076	2407.273	12592.56
[illegible]			*8695.503	*1449.291	2427.521	12572.31
[illegible]			*8660.201	*1443.407	2448.113	12551.72
[illegible]			*8624.295	*1437.423	2469.057	12530.77
[illegible]			*8587.770	*1431.336	2490.363	12509.47
[illegible]			*8550.600	*1425.143	2512.040	12487.79
[illegible]			*8512.794	*1418.841	2534.097	12465.73
[illegible]			*8474.310	*1412.427	2556.545	12443.28
[illegible]			*8435.138	*1405.899	2579.395	12420.43
[illegible]			*8395.259	*1399.253	2602.656	12397.16
14.09[illegible]			*8354.655	*1392.486	2626.341	12373.48
[illegible]			*8313.305	*1385.595	2650.461	12349.36
[illegible]			*8271.188	*1378.575	2675.028	12324.79
[illegible]			*8228.283	*1371.425	2700.055	12299.76
[illegible]			*8184.568	*1364.140	2725.554	12274.26
[illegible]			*8140.019	*1356.715	2751.540	12248.27
[illegible]			*8094.613	*1349.148	2778.026	12221.78
[illegible]			*8048.324	*1341.434	2805.027	12194.78
[illegible]			*8001.126	*1333.568	2832.558	12167.25
[illegible]			*7952.993	*1325.546	2860.634	12139.17
[illegible]			*7903.896	*1317.364	2889.273	12110.53
[illegible]			*7853.806	*1309.016	2918.491	12081.31
[illegible]			*7802.692	*1300.497	2948.306	12051.49
[illegible]			*7750.524	*1291.803	2978.736	12021.06
16.15[illegible]			*7697.267	*1282.928	3009.801	11989.99
[illegible]			*7642.888	*1273.865	3041.521	11958.27
16.49669			*7587.350	*1264.609	3073.917	11925.87
16.67429			*7530.617	*1255.154	3107.010	11892.78
16.85575			*7472.648	*1245.493	3140.824	11858.96
17.04101			*7413.404	*1235.620	3175.381	11824.40
17.23070			*7352.843	*1225.527	3210.708	11789.07
17.42465			*7290.917	*1215.207	3246.829	11752.95
17.62251			*7227.583	*1204.652	3283.772	11716.00
17.82577			*7162.791	*1193.853	3321.566	11678.21
18.03[illegible]			*7096.491	*1182.804	3360.240	11639.53
18.24572			*7028.629	*1171.494	3399.825	11599.94

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18.46323	*	*6959.147	*1159.915	3440.353	11559.41
18.68598	*	*6887.990	*1148.056	3481.860	11517.90
18.91417	*	*6815.095	*1135.907	3524.380	11475.38
19.14801	*	*6740.397	*1123.459	3567.952	11431.80
19.38770	*	*6663.830	*1110.698	3612.615	11387.14
19.63346	*	*6585.321	*1097.614	3658.410	11341.34
19.88554	*	*6504.796	*1084.194	3705.381	11294.37
20.14417	*	*6422.176	*1070.425	3753.573	11246.17
20.40962	*	*6337.380	*1056.293	3803.036	11196.70
20.68216	*	*6250.318	*1041.783	3853.820	11145.92
20.96208	*	*6160.900	*1026.881	3905.978	11093.76
21.24968	*	*6069.028	*1011.570	3959.568	11040.16
21.54528	*	*5974.600	*995.8336	4014.649	10985.08
21.84922	*	*5877.508	*979.6526	4071.283	10928.44
22.16185	*	*5777.638	*963.0085	4129.539	10870.18
22.48357	*	*5674.867	*945.8811	4189.485	10810.23
22.81476	*	*5569.070	*928.2492	4251.198	10748.51
23.15586	*	*5460.108	*910.0900	4314.756	10684.95
23.50731	*	*5347.840	*891.3796	4380.244	10619.46
23.86959	*	*5232.110	*872.0925	4447.750	10551.95
24.24321	*	*5112.758	*852.2017	4517.369	10482.32
24.62872	*	*4989.610	*831.6782	4589.202	10410.49
25.02668	*	*4862.482	*810.4915	4663.357	10336.33
25.43772	*	*4731.179	*788.6088	4739.948	10259.73
25.86248	*	*4595.490	*765.9953	4819.096	10180.58
26.30167	*	*4455.193	*742.6138	4900.933	10098.74
26.75603	*	*4310.048	*718.4245	4985.597	10014.07
27.22637	*	*4159.801	*693.3847	5073.237	9926.423
27.71354	*	*4004.177	*667.4489	5164.014	9835.640
28.21846	*	*3842.882	*640.5679	5258.099	9741.549
28.74213	*	*3675.600	*612.6893	5355.676	9643.966
29.28559	*	*3501.993	*583.7564	5456.943	9542.693
29.85000	*	*3321.694	*553.7083	5562.113	9437.515
30.43660	*	*3134.308	*522.4792	5671.417	9328.204
31.04671	*	*2939.410	*489.9982	5785.102	9214.511
31.68179	*	*2736.539	*456.1882	5903.439	9096.167
32.34338	*	*2525.195	*420.9662	6026.718	8972.879
33.03320	*	*2304.835	*384.2418	6155.256	8844.333
33.75309	*	*2074.871	*345.9167	6289.396	8710.184
34.50505	*	*1834.661	*305.8840	6429.512	8570.058
35.29128	*	*1583.504	*264.0270	6576.015	8423.546
36.11417	*	*1320.634	*220.2179	6729.349	8270.202
36.97635	*	*1045.213	*174.3171	6890.004	8109.535
37.88071	*	*756.3203	*126.1710	7058.519	7941.010
38.83042	*	*452.9412	*75.61079	7235.482	7764.034
39.82897	*	*133.9589	*22.45018	7421.548	7577.957
40.88023	*	*-201.862	*-33.5167	7617.435	7382.056

CALCULATION OF STATIC MARGINS FOR SAMPLE MISSIONS

Calculation of Center of Pressure, xcp, with 5000 lb Return Payload

W	12145.05
CD	1.766044
B	2.252975
DIA	62.34128
l	33.17110
xcp	-5.49291

r = distance from reference point to center of mass of section
m = mass of section
xcm = (totl rxm)/(totl m)
static margin = xcm - xcp

	r	m	r x m
O2 tank	28	444.21	12437.88
H2 tank	17.5	342.84	5999.7
N2 tank	8	358	2864
arobrke	43	2300	98900
structure	21	1840	38640
avionics	34.5	930	32085
att cntrl	21.5	200	4300
proplsn	39.5	530	20935
docking	1.5	200	300
payload	0	0	0

totl m	totl rxm	xcm
7145.05	216461.6	30.29532

static margin
****
35.78823

	r	m	r x m
O2 tank	28	444.21	12437.88
H2 tank	17.5	342.84	5999.7
N2 tank	8	358	2864
arobrke	43	2300	98900
structure	21	1840	38640
avionics	34.5	930	32085
att cntrl	21.5	200	4300
proplsn	39.5	530	20935
docking	1.5	200	300
payload	-6	4000	-24000

totl m	totl rxm	xcm
11145.05	192461.6	17.26879

static margin
****
22.76170

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	r	m	r x m
O2 tank	28	444.21	12437.88
H2 tank	17.5	342.84	5999.7
N2 tank	8	358	2864
arobrke	43	2300	98900
structure	21	1840	38640
avionics	34.5	930	32085
att cntrl	21.5	200	4300
proplsn	39.5	530	20935
docking	1.5	200	300
payload	-8	11000	-88000

totl m	totl rxm	xcm
18145.05	128461.6	7.079704

static margin
****
12.57261

CALCULATION OF HYDROSTATIC PRESSURE IN HYDROGEN TANK DUE TO ACCELERATION

VOLUME	ACC	H	P	GAMMA	DENSITY H2	PAYLOAD 15000	ACTUAL H
FULL	7.584	1921.738	37.35289	.0194370	.0025629		172.5
3/4	9.045	1595.991	36.99736	.0231814	.0025629		129.3
1/2	11.45	1244.822	36.52956	.0293452	.0025629		86.2
1/4	15.3	910.1733	35.69005	.0392124	.0025629		43.1

PROPELLANT TANK PRESSURIZATION CALCULATION

Values for Nitrogen

gas constant, R = 55.15 ft lbf/lbm R
specific heat ratio, γ = 1.4
initial tank temperature, T_o = 450.6 R
initial tank pressure, P_o = 2205 psia
minimum tank pressure, P_{min} = 573.3 psia
density at P_o and T_o, ρ_o = 12.7952 lbm/ft

Values for Oxygen

tank volume, V_{O_2} = 572.098 ft
tank pressure, P_{O_2} = 22 psia

Values for Hydrogen

tank volume, V_{H_2} = 1553.845 ft
tank pressure, P_{H_2} = 34 psia

Mass of Nitrogen Needed to Pressurize Oxygen Tank

$$m = (P_{O_2} V_{O_2} / R\, T_o)\{\gamma / [1 - (P_{min}/P_o)]\}$$

$$m = \frac{(22)(144)(572.098)}{(55.15)(450.6)} \times \frac{1.4}{1 - (573.3/2205)}$$

m = 137.98 lbm

Mass of Nitrogen Needed to Pressurize Hydrogen Tank

$$m = (P_{N_2} V_{N_2} / R\, T_o)\{\gamma / [1 - (P_{min}/P_o)]\}$$

$$m = \frac{(34)(144)(1553.845)}{(55.15)(450.6)} \times \frac{1.4}{1 - (573.3/2205)}$$

m = 579.174 lbm

Total Nitrogen Needed to Pressurize Propellants

m = (137.98 + 579.174)lbm = 717.154 lbm

volume, $V_{N_2} = m/\rho_o$ = (17.154 lbm)/(12.7952 lbm/ft)

V_{N_2} = 56.0487 ft^3